NARASARAOPETA ENGINEERING COLLEGE:: (AUTONOMOUS) DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING <u>Network Analysis</u>

BASIC LAWS:





 $E_1-E_2=V_1-V_2+V_3-V_4=I_1Z_1-I_2Z_2+I_3Z_3-I_4Z_4$

Linear and Nonlinear Networks

A network is linear if the principle of superposition holds i.e. if e1 (t), r1 (t) and e2 (t), r2 (t) are excitation and response pairs then if excitation is e1 (t) + e2 (t) then the response is r1 (t) + r2(t).

The network not satisfying this condition is nonlinear Ex: - Linear – Resistors, Inductors, Capacitors.

Nonlinear – Semiconductors devices like transistors, saturated iron core inductor, capacitance of a p-n function.

Passive and active Networks:

A Linear network is passive if (i) the energy delivered to the network is nonnegative for any excitation. (ii) No voltages and currents appear between any two terminals before any excitation is applied.

Example: - R, L and C.

Active network: - Networks are containing devices having internal energy – Generators, amplifiers and oscillators.

Unilateral & Bilateral:

The circuit, in which voltage current relationship remains unaltered with the reversal of polarities of the source, is said to be bilateral.

Ex: - R, L & C

If V-I relationships are different with the reversal of polarities of the source, the circuit is said to be unilateral. Ex: - semiconductor diodes.

Lumped & Distributed:

Elements of a circuit, which are separated physically, are known as lumped elements. Ex: - L & C.

Elements, which are not separable for analytical purposes, are known as distributed elements. Ex:- transmission lines having R, L, C all along their length.

Delta-star transformation:

A to Y transformation:

Consider three \Box -connected impedances ZAB, ZBC and ZCA across terminals A, B and C. It is required to replace these by an equivalent set ZA, ZB and ZC connected in star.



$$Z_{A} = \frac{Z_{CA} Z_{AB}}{Z_{AB} + Z_{B} + Z_{CA}} \qquad Z_{B} = \frac{Z_{AB} Z_{BC}}{\sum Z_{AB}} \quad Z_{C} = \frac{Z_{BC} Z_{CA}}{\sum Z_{AB}}$$

If
$$Z_{AB} = Z_{BC} = Z_{CA} = Z_{\Delta}$$
 then $Z_A = Z_B = Z_C = Z_Y = \frac{Z_{\Delta}}{3}$

Y to ▲ transformation:

Consider three Y connected admittance Y_a , Y_b and Y_c across the terminals A, B and C. It is required to replace them by a set of equivalent Δ admittances Y_{ab} , Y_{bc} and Y_{ca} .

Similarly
$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$$
 $Z_{CA} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B}$

Network Topology

Definition

The term circuit topology refers to the science of placement of elements and is a study of the geometric configurations. "Circuit topology is the study of geometric properties of a circuit useful for describing the circuit behaviour" **Graph**:

In the given network if all the branches are represented by line segments then the resulting figure is called the graph of a network (or linear graph). The internal impedance of an ideal voltage source is zero and hence it is replaced by a short circuit and that of an ideal current source is infinity and hence it is represented by an open circuit in the graph.

Node

It is a point in the network at which two or more circuit elements are joined. In the graph shown 1, 2, 3 and 4 are nodes.

Branch (or Twig):

It is a path directly joining two nodes. There may be several parallel paths between two nodes.

Oriented Graph

If directions of currents are marked in all the branches of a graph then it is called an oriented (or directed) graph

Connected graph

A network graph is connected if there is a path between any two nodes .In our further discussion, let us assume that the graph is connected. Since, if it is not connected each disjoint part may be analysed separately as a connected graph.

Unconnected graph

If there is no path between any two nodes, then the graph is called an unconnected graph.

Planar graph

A planar graph is a graph drawn on a two dimensional plane so that no two branches intersect at a point which is not a node.

Non - planar graph

A graph on a two – dimensional planes such that two or more branches intersect at a point other than node on a graph.

Tree of a graph

Tree is a set of branches with all nodes not forming any loop or closed path. (*) Contains all the nodes of the given network or all the nodes of the graph (*) No closed path

(*) Number of branches in a tree = n-1, where n=number of nodes

Co- tree

A Co- tree is a set of branches which are removed so as to form a tree or in other words, a co- tree is a set of branches which when added to the tree gives the complete graph. Each branch so removed is called a link.

Number of links = I = b - (n-1) where b = Total number of branches

Incidence Matrix

Incidence matrix is a matrix representation to show which branches are connected to which nodes and what is their orientation in a given graph

(*) The rows of the matrix represent the nodes and the columns represents the branches of the graph.

(*) The elements of the incidence matrix will be +1, -1 or zero (*) If a branch is connected to a node and its orientation is away from the node the corresponding element is marked +1

(*) If a branch is connected to a node and its orientation is towards the node then the corresponding element is marked – 1

(*) If a branch is not connected to a given node then the corresponding element is marked zero.

(i)Complete incidence matrix:

An incidence matrix in which the summation of elements in any column in zero is called

a complete incidence matrix.

(ii)Reduced incidence matrix:

The reduced incidence matrix is obtained from a complete incidence matrix by eliminating a row. Hence the summation of elements in any column is not zero.

Tie - set Analysis:

In order to form a tree from a network several branches need to be removed so that the closed loops open up. All such removed branches are called links and they form a Co-tree.

Alternatively when a link is replaced in a tree, it forms a closed loop along with few of the tree branches. A current can flow around this closed loop. The direction of the loop current is assumed to be the same as that of the current in the link. The tree – branches and the link that form a loop is said to constitute a tie – set.

Cut - set Analysis

A cut – set of a graph is a set of branches whose removal , cuts the connected graph in to two parts such that the replacement of any one branch of the cut set renders the two parts connected

Formation of Fundamental cut - set

Select a tree

(*) Select a tree branch

(*) Divide the graph in to two sets of nodes by drawing a dotted line through the selected tree branch and appropriate links while avoiding interruption with any other tree – branches **DUALITY CONCEPT** Two electrical networks are duals if the mesh equations that characterize one have the same mathematical form as the nodal equations of the other.

Table of dual Quantities

Current source
Node voltages
Capacitances
Conductance
Inductances
On KCL basis
Opening of switch

SUPERPOSITION THEOREM:

The response of a linear network with a number of excitations applied simultaneously is equal to the sum of the responses of the network when each excitation is applied individually replacing all other excitations by their internal impedances

Reciprocity Theorem:

In an initially relaxed linear network containing one independent source only. The ratio of the response to the excitation is invariant to an interchange of the position of the excitation and the response.

Millman's Theorem:

Certain simple combinations of potential and current source equivalents are of use because they offer simplification in solutions of more extensive networks in which combinations occur. Millman's Theorem says that "if a number of voltage sources with internal impedances are connected in parallel across two terminals, then the entire combination can be replaced by a single voltage source in series with single impedance".

Thevenin's Theorem :

If two linear networks one M with passive elements and sources and the other N with passive elements only and there is no magnetic coupling between M and N, are connected together at terminals A and B, then with respect to terminals A and B, the network M can be replaced by an equivalent network comprising a single voltage source in series with a single impedance. The single voltage source is the open circuit voltage across the terminals A and B and single series impedance is the impedance of the network M as viewed from A and B with independent voltage sources short circuited and independent current sources open circuited. Dependent sources if any are to be retained.



$$I_L = \frac{V_{TH}}{Z_{TH} + Z_L}$$

Norton's Theorem:-

The Thevenin's equivalent consists of a voltage source and a series impedance. If the circuit is transformed to its equivalent current source, we get Norton's equivalent. Thus Norton's theorem is the dual of the Thevenin's theorem.



Maximum Transfer Theorem:-

When a linear network containing sources and passive elements is connected at terminals A and B to a passive linear network, maximum power is transferred to the passive network when its impedance becomes the complex conjugate of the Thevenin's impedance of the source containing network as viewed form the terminals A and B.



Resonant Circuits

Resonance is an important phenomenon which may occur in circuits containing both inductors and capacitors.

The resonance circuits can be classified in to two categories

Series – Resonance Circuits. Parallel – Resonance Circuits.

1. Series Resonance Circuit

In other words, by varying the frequency it is possible to reach a point where XL = XC. In that case Z = R and hence circuit will be under resonance. Hence the series A.C. circuit is to be under resonance, when inductive reactance of the circuit is equal to the capacitive reactance. The frequency at which the resonance occurs is called as resonant frequency (fr)

Expression for Resonant Frequency (fr) At resonance XL = XC

Salient Features of Resonant circuit

(*) At resonance XL = XC

(*) At resonance Z = R i.e. impedance is minimum and hence I = V/Z is maximum

(*) The current at resonance (Ir) is in phase with the voltage

(*) The circuit power factor is unity

(*) Voltage across the capacitor is equal and opposite to the voltage across the inductor.

Selectivity:

Selectivity is a useful characteristic of the resonant circuit. Selectivity is defined as the ratio of band width to resonant frequency Selectivity = f2-f1

Parallel Resonance

A parallel resonant circuit is one in which a coil and a capacitance are connected in parallel across a variable frequency A.C. Supply. The response of a parallel resonant circuit is somewhat different from that of a series resonant circuit.

R-L Series circuit transient:

Consider The R-L series circuit shown in the fig. Switch K is closed at t=0. Referring to the circuit, balance equation using

$$\vee$$
(t) = R_i(t) + $\frac{\text{Idi}(t)}{\text{dt}}$

Kirchhoff's law can be written as Taking Laplace Transform we get

$$\frac{\forall (s)}{s} = I(s) \times R + L\{SI(s) - I(0)\}$$

$$i(t) = \frac{V}{R} \left[1 - e^{-\left(\frac{R}{L}\right)t} \right]$$

R-C series circuit Transient



Consider the RC circuit shown. Let the switch be closed at t=0.

Writing the balance equation using Kirchoff"s voltage law ,

$$i(t) = \frac{V(t)}{R} e^{-\left(\frac{1}{RC}\right)t}$$



Assuming zero initial conditions when switch K is closed the balanced equation is given by

$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int idt$$

Taking Laplace transformation we get

$$\frac{V(s)}{s} = 1(s)R + LSI(s) + \frac{I(s)}{CS}$$

The time response of the circuit depends on the poles or roots of the characteristic equation

$$S^2 + S\frac{R}{L} + \frac{1}{LC} = 0$$

Roots of the characteristic equation are given by

$$S_{1}, S_{2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^{2} - 4 \times \frac{1}{LC}}}{2}$$
$$S_{1}, S_{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^{2} - \frac{1}{LC}}$$

TWO PORT PARAMETERS:



Multi port

PORT:- Pair of terminals at which an electrical signal enters or leaves a network.

One port network: - Network having only one port.

Ex: Domestic appliances, Motor, Generator, Thevenin's or Norton networks **Two port network**: - Network having an input port and an output port. Ex: Amplifiers, Transistors, communication circuits, Power transmission & distribution lines

Filters, attenuators, transformers etc.

Multi-port network:-Network having more than two ports.

Ex: Power Transmission lines, Distributions Lines, Communication lines.

Four important Parameters



S1.	Parameters	Dependent	Independent	Equations
No.		Variable	Variable	
1.	z Parameters	V ₁ , V ₂	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
2.	y parameters	I_1, I_2	V ₁ , V ₂	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
3.	h parameters	V ₁ , I ₂	I ₁ , V ₂	$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$
4.	t parameters	V_1, I_1	V_2, I_2	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$

For reciprocity with $z_{12}=z_{21}$,

In terms of y parameters $z_{12} = -y_{12}/[]y \& z_{21} = -y_{21}/[]y \text{ condition is } y_{12} = y_{21}$

In terms of h parameters $z_{12} = h_{12}/h_{22} \& z_{21} = -h_{21}/h_{22}$ the condition is $h_{12} = -h_{21}$ In terms of t parameters $z_{12} = [t/C \& z_{21} = 1/C$ the condition is [t=AD - BC=1]

SYMMETRICAL CONDITIONS

A two port is said to be symmetrical if the ports can be interchanged without changing the port voltage and currents..

As $z_{11}=z_{22}$, in terms of y we have $y_{11}=z_{12}/dz & y_{22}=z_{1}/dz$, In terms of t parameters as $z_1=A/C & z_{22}=D/C$ the condition is A=D